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Advanced Quantum Information Theory

Introduction & Course Outline

- Will only need very basic quant-inf. knowledge
- Lecture notes; examples sheets

Outline:

QIT course, "preliminaries" link, Nielsen + Chuang

0. Background (Complexity Theory)
   Kitaev, Shen, Vyalyi – everything needed here
   Nielsen + Chuang – almost everything (no QMA)
   Arora + Barak – to go further (classical)

I. Hamiltonian Complexity
   Closely follow Kitaev book

II. Many-body quantum systems
   Research papers, Hastings' Les Houches notes

III. Hamiltonian Simulation Theory
   [OT'08] [BDL'11] [CMP'19]

Notation and Terminology

Basic:

$\bar{z}$: complex conjugate of $z \in \mathbb{C}$

$[a,b]$: closed interval a to b in $\mathbb{R}$, i.e. $[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

$\mathbb{C}^2$: Hilbert space of dim 2 ("qubit")

$\mathbb{C}^d$: d-dim Hilbert space ("qudit")

$\text{span}\{|\psi_i\rangle\}$: linear subspace spanned by $|\psi_1\rangle, |\psi_2\rangle, \ldots$

$$\mathcal{H}^{\otimes n} \equiv \bigotimes_{i=1}^{n} \mathcal{H}$$ e.g. n-qubits: $(\mathbb{C}^2)^{\otimes n}$

$\pi^{(0)} = |0\rangle\langle 0|$, $\pi^{(1)} = |1\rangle\langle 1|$ (orthog. projectors)

Big-O notation:

$f(x) = \mathcal{O}(g(x))$
$\Leftrightarrow \exists\, c, n > 0$ s.t. $\forall x > n$: $f(x) < c\,g(x)$

$f(x) = \Omega(g(x))$
$\Leftrightarrow \exists\, c, n > 0$ s.t. $\forall x > n$: $f(x) > c\,g(x)$

Operators:

$\mathcal{B}(\mathcal{H})$: set (actually, algebra) of all bounded operators on Hilbert space $\mathcal{H}$

(Recall: X bounded $\Leftrightarrow$ $\| X|\psi\rangle \| < \infty$.
<u>All</u> linear operators on finite-dim $\mathcal{H}$ bounded)

$\mathcal{M}_n$: $n \times n$ complex matrices

Herm_n : $n \times n$ Hermitian matrices
$H \in \mathrm{Herm}_n \Leftrightarrow H^\dagger = H$

$\mathrm{Herm}(\mathcal{H})$: Hermitian operators on Hilbert space $\mathcal{H}$

$\mathbb{1}_d$: d-dimensional identity matrix

$\bar{M}$: entry-wise complex conjugation of $M \in \mathcal{M}_n$.

$\mathrm{Re}(X)$, $\mathrm{Im}(X)$: real, imag parts of X (entry-wise for $X \in \mathcal{M}_d$)

$\mathrm{spec}(M)$: spectrum of M (i.e. set of eigenvalues) <u>not</u> including degeneracies

$$M^{\oplus n} := \underbrace{M \oplus M \oplus \cdots \oplus M}_{n \text{ times}}$$

$\ker A := \mathrm{span}\{|\psi\rangle : A|\psi\rangle = 0\}$ (kernel)

$\mathrm{supp}\, A := (\ker A)^{\perp}$ ("support")

Warning: QIT terminoloy! "co-image" elsewhere

positive operator $H > 0$ ($\Rightarrow H^{\dagger} = H$)

$\Leftrightarrow \forall |\psi\rangle \quad \langle\psi| H |\psi\rangle > 0$

positive semi-definite: replace '$>$' with '$\geq$'